Review

Laser Doppler Anemometry — a Non-Intrusive Optical Measuring Technique for Fluid Velocity

Bodo Ruck *

(Received: 22 August 1986)

Abstract

The present paper gives an introduction to laser Doppler anemometry. The fundamental principles of this measuring technique, as well as the basic features of its instrumental realizations, are explained. The application of laser Doppler anemometry to fluid investigations often requires an adaptation of the LDA-system to the measuring problem. Basic formulas for the lay-out and adaptation are given.

1 Introduction

For many investigations of technical and natural processes the measurement of fluid velocity is inevitable because many quantities, for example the determination of volume and mass fluxes, are based on accurate velocity measurements. In addition to single-phase flows, an appropriate fluid velocity measuring technique is required for the assessment of laminar and turbulent two-phase flows and dispersion and transport processes. The laser invention in 1960 provided the opportunity of measuring fluid velocity by the detection of the beat signals of two coherent light waves which are scattered on a moving interface within a fluid. This measuring technique was soon used to measure the velocity of very small suspended particles in liquid flows (Yeh et al. [1]). Provided the particle diameter is small enough, the particles will follow the flow fluctuations and the measuring technique can be used to determine the fluid velocity. By analogy to the acoustical Doppler effect, which also describes the same measuring principle analytically, the terminology “laser Doppler anemometry” arose. The advantages of this measuring technique are apparent. On one hand the laser beam properties such as its focusability result in a high spatial resolution, on the other hand processes are not disturbed by probes intruding into the flow. Furthermore, LDA-measuring systems need no calibration since the principle is based on physical constants such as the velocity of light.

Today laser Doppler anemometry is a common fluid velocity measuring technique which is extensively used in research and development as well as in the control of industrial production processes. Fundamental information about the measuring technique can be found in Rudd [2], Farmer [3], Eliasson et al. [4], Ouest et al. [5], Drain [6], Ruck [7]. In the following sections the basic features of laser anemometry are explained. New results concerning the realization and applicability of LDA-systems are also given.

2 Fundamental Principles

A laser Doppler anemometer, sometimes called a laser Doppler velocimeter, represents an optical measuring system for local, non-intrusive velocity measurements in fluids which is normally divided into three parts, the light source, the emission optics and the detection optics. Usually continuous wave lasers in the milliwatt power range, working in the TEM₀₀—mode, are used in laser Doppler anemometry. The LDA principle is based on the physical fact that coherent light waves, which are scattered by moving interfaces, show a Doppler shift and hence contain velocity information.

In the case of a fluid, suspended particles, droplets, natural impurities and also small bubbles can be used as the scattering medium. If the scattering centres are sufficiently small, they will follow the flow ideally which infers that the velocity of the scattering phase is the same as that of the continuous phase.

The frequency shift of laser light wave being scattered by a moving particle can be described by the Doppler formula

\[
\nu' = \nu_0 \left[ 1 - \frac{\vec{u} \cdot \vec{l}}{c} \right].
\]

(1)

In Eq. (1) \( \nu_0 \) denotes the light frequency, \( \vec{u} \cdot \vec{l} \) represents the scalar product of the particle velocity vector and the vector indicating the direction of propagation of the illuminating laser beam, \( c \) is the velocity of light and \( \nu' \) the light frequency seen by the particle. A detector in space, direction vector \( \vec{l} \) detects...
\[ f_D = f_0 \frac{1 - \frac{\tilde{u} l}{c}}{1 - \frac{\tilde{u} l_D}{c}} \]  

which can be written in series form:

\[ f_D = f_0 \left( 1 - \frac{\tilde{u} l}{c} + \frac{\tilde{u} l_D}{c} \right). \]  

As can be seen, Eq. (3) contains the velocity information. Unfortunately, the frequency range is about $10^{14} - 10^{15}$ Hertz, which is in the range of the original light frequency. Signal frequencies of this order can hardly be resolved with sufficient accuracy. In laser Doppler anemometry, therefore, the light beam of the laser is split into two partial beams (see Figure 1). The two partial beams are focused by means of the convex lens and form at its intersection point the measuring volume. When using two laser beams the detector registers the superposition of two separate Doppler effects as given in Eq. (3).

![Diagram of a LDA two beam anemometer](image)

**Fig. 1:** Principle of a LDA two beam anemometer.

The resulting scalar products, see Eq. (1), are not identical because the direction vectors, \( l_1 \) and \( l_2 \), of the partial beams are different. This results in two different, Doppler shifted light waves with frequencies \( f_{01} \) and \( f_{02} \). It is known, from scalar wave theory, that light with frequencies \( f_1 (\lambda_1) \) and \( f_2 (\lambda_2) \) can be simplified to denote the electric field as:

\[ E_1 = E_0 \cos 2\pi \left( \frac{f_1 t}{\lambda_1} - \frac{x}{\lambda_1} \right) \]

\[ E_2 = E_0 \cos 2\pi \left( \frac{f_2 t}{\lambda_2} - \frac{x}{\lambda_2} \right). \]

Superposition yields a light wave which is given by:

\[ E_1 = 2E_0 \cos 2\pi \left( \frac{f_1 + f_2}{2} t - \frac{\lambda_1 + \lambda_2}{2\lambda_1 \lambda_2} x \right) \]

\[ \cos 2\pi \left( \frac{f_1 - f_2}{2} t - \frac{\lambda_1 - \lambda_2}{2\lambda_1 \lambda_2} x \right). \]

It can be inferred from Eq. (6) that the resulting signal consists of a high frequency wave which is modulated by a low frequency beat, \( \Delta f = f_1 - f_2 \). If we now consider this relationship we observe a beat frequency, \( \Delta f = f_{01} - f_{02} \), which is both easy to resolve (frequency range $10^6$ Hz) and also does not show any dependence on the detection direction. This beat frequency, \( \Delta f \), is usually slightly incorrectly referred to as the Doppler frequency:

\[ \Delta f = f_{01} - f_{02} = f_0 \left[ \frac{\tilde{u} l}{c} - \frac{\tilde{u} l_D}{c} \right]. \]

Rewriting the scalar products, using the nomenclature in Figure 1 and taking \( f_0/c = 1/\lambda \) yields:

\[ \Delta f = \frac{|\tilde{u}|}{\lambda} \left[ \cos (\psi - \varphi) - \cos (\psi + \varphi) \right]. \]

A trigonometrical transformation simplifies this relation between the detection frequency and the measured velocity component

\[ \Delta f = \frac{|\tilde{u}|}{\lambda} \sin \psi \frac{2 \sin \varphi}{\lambda}. \]

Eqs. (7) and (9) express the fact that the frequency which is measured is proportional to the velocity component \( |\tilde{u}| \sin \psi \), that is component perpendicular to the bisector of the laser beam directions \( l_1 \) and \( l_2 \).

So far the derivations of the principle have been restricted to the acoustical Doppler effect which differentiates between emitter and detector. It is known that the theory of relativity denies this difference.

Following this theory, one must use Eq. (1) twice for the emission and detection directions. This leads to a change in sign in the last term in the bracket of Eq. (3). This term describes the dependence of the shifted light frequency on the detection direction. Forming the difference \( \Delta f = f_{01} (\tilde{u}, l_1, l_D) - f_{02} (\tilde{u}, l_2, l_D) \) of two Doppler shifted light waves leads to the elimination of this term, irrespective of sign. Hence we deduce the independence of the beat frequency on the detection direction and derive the same result as by acoustical Doppler considerations.

Because of simplicity and a better graphical treatment, the acoustical Doppler explanations are usually preferred.

In engineering another description model, the fringe model, is very often used. The model postulates the existence of interference fringes at the intersection point, whose spacing, \( \Delta x \), is

![Diagram of fringe model](image)

**Fig. 2:** Derivation of the fringe model in laser Doppler anemometry – superposition of two laser beams.
a function of the semi-angle between the two laser beams and of the light wavelength, \( \lambda \). To derive this descriptive model we must consider the superposition of two laser beams which have plane wave fronts. Figure 2 shows a sketch of the measuring volume formed by the laser beams. The superposition of two waves with identical frequencies can be described by the summation of both single waves, which can be simply represented by:

\[
E_i = E_0 \cos(\omega t - ky_i)
\]  

(10)

\[
E_x = E_0 \cos(\omega t - ky_x)
\]  

(11)

with \( \omega = 2\pi f \) and \( k = 2\pi/\lambda \).

The sum of two cosine terms can, trigonometrically, be rewritten as the product of two cosine terms with the difference or sum of the original arguments.

\[
E = 2E_0 \cos\left(\frac{ky_2 - ky_1}{2}\right) \cos\left(\frac{2\omega t - ky_2 - ky_1}{2}\right).
\]  

(12)

The intensity can be computed hereafter using

\[
I = \lim_{T \to \infty} \frac{1}{T} \int_0^T E^2 \, dt.
\]  

(13)

Squaring and substitution of \( \frac{2\omega t - ky_2 - ky_1}{2} = w \) leads with

\[
I = \frac{4E_0^2}{\omega T} \cos^2\left(\frac{ky_2 - ky_1}{2}\right) \int_0^\infty \cos^2 \theta \, d\theta.
\]  

(14)

to the expression

\[
I = 2E_0^2 \cos^2\left(\frac{ky_2 - ky_1}{2}\right) \left[ 1 + \frac{1}{2\omega T} (\sin \cdots) \right].
\]  

(15)

An appraisal for \( T \gg 1/f \) gives the time-independent, loci-dependent relationship

\[
I = 2E_0^2 \cos^2\left(\frac{ky_2 - ky_1}{2}\right).
\]  

(16)

A simple coordinate transformation for \( y_1 \) and \( y_2 \) yields

\[
y_1 = x_1 \cos \varphi - x_2 \sin \varphi
\]  

(17)

\[
y_2 = x_1 \cos \varphi + x_2 \sin \varphi.
\]  

(18)

The subtraction of both equations yields:

\[
y_2 - y_1 = 2x_2 \sin \varphi.
\]  

(19)

The intensity maxima in Eq. (16) occur as a function of local coordinates, if the argument of the cosine function is a multiple of \( \pi \). Therefore, for the fringe numbers \( n \) and \( n + 1 \) the following equations hold:

\[
\pi 2x_{2,n} \sin \varphi = (n + 1) \lambda \pi
\]  

(20)

Subtraction of Eq. (20) from Eq. (21) shows the fringe spacing to be

\[
\Delta x = x_{2,n} - x_{2,n+1} = \frac{\lambda}{2\sin \varphi}.
\]  

(22)

The concept of the fringe model is that a particle suspended in a fluid moves through the measuring volume and consecutively scatters the bright and dark sections of the interference pattern. A detector located in space detects these frequencies, which correspond to the velocity component perpendicular to the fringe system, \( u_z \).

\[
\Delta f = \frac{u_z}{\Delta x} = u_z \frac{2\sin \varphi}{\lambda}
\]  

(23)

rewritten

\[
u_z = \frac{\Delta f \lambda}{2\sin \varphi}.
\]  

(24)

It can easily be seen that Eq. (23) and Eq. (9) are identical. The fringe spacing, \( \Delta x \), is an important characteristic of a LDA system. Furthermore, it can be seen that all existing description models in laser Doppler anemometry (Drain [6], Ruck [8]) finally lead to the same formula. If the semi-angle between the crossing laser beams and the wave length of the laser light is fixed, it is easy to determine from Eqs. (23) and (24) the beat frequency corresponding to unit fluid velocity. The linear proportionality between velocity and frequency distinguishes the LDA method from other existing fluid velocity measuring methods.

Figure 3 shows a typical LDA signal as can be detected by a storage oscilloscope. The LDA burst contains the signal frequency, that is the beat frequency of both Doppler shifted light waves. The signal envelope is caused by the Gaussian intensity profiles of the laser beams.

**Fig. 3:** Typical LDA signal; modulation depth \( \eta \).

The signal quality can be assessed by the modulation depth, \( \eta \), which is often referred to as „visibility“. The visibility depends normally on a variety of parameters. The most important parameters of influence are particle diameter, type of optical system, background noise and number of particles in the measuring volume.

Signal detection in laser Doppler anemometry is normally based on photomultipliers or photodiodes. The detection elements must be selected according to the power requirements and with an adequate frequency range (Durst et al. [9], Ruck et al. [10]). The measuring volume should be properly imaged on the pinhole of the photodetector. It should be ensured that the photocathode detects only the scattered light which comes from the intersection region of both illuminating laser beams. The
laser beams themselves must be blocked to avoid direct illumination of the photoelement. The dimensions of the pinhole aperture placed before the detector cathode can be computed by the laws of geometrical optics taking into account the focal length and the measuring volume extension (see section 3).

2.1 LDA-Arrangements

The previous sections are based on the dual beam anemometer which is characterized by two laser beams of equal intensity forming the measuring volume. The derivations of the LDA-principle can also be extended to systems with unequal intensity in both illuminating beams. If we introduce different amplitudes, \( E_{\text{O,A}} \) and \( E_{\text{O,B}} \), into Eqs. (4) and (5), (10) and (11) we obtain, after a similar derivation the following expression for the resultant intensity in the measuring volume:

\[
I = \frac{E_{\text{O,A}}^2}{2} + \frac{E_{\text{O,B}}^2}{2} + E_{\text{O,A}} E_{\text{O,B}} \cos(ky_2 - ky_1).
\]  

(25)

It can easily be seen from Eq. (25) that the modulation of the intensity is a maximum only for \( E_{\text{O,A}} = E_{\text{O,B}} \). For all other cases the visibility decreases but the frequencies is not affected. Since the LDA-principle is frequency-dependent, the velocity information is, therefore, not influenced by different intensities in the laser beams. Hence, other arrangements were used in the past to realize the LDA-principle. Figure 4 is a compilation of the most commonly used LDA-arrangements.

![Diagram of LDA-arrangements](image)

Fig. 4: Common LDA-arrangements.

2.2 Single and Multiple Particle Signals

The application of laser Doppler anemometry is based on a single particle evaluation. Occasionally the particle number concentration is so high that more than one particle resides in the measuring volume at the same time. It is useful to discuss the resulting influence on the velocity information. If we consider two particles passing the measuring volume at the same time, without velocity difference, the distance between the particles and the particle size remain as parameters of influence (Farmer [3]). It should be clarified, whether the signal information from both particles can combine destructively or even can bias the velocity information. To anticipate the answer, the LDA-frequency is not affected by multiple particle signals.

The most unfavourable case occurs if two particles of identical size, form and consistency pass a measuring volume whose fringe spacing is \( \Delta x \) with a separation of \( (\Delta x/2 + n \cdot \Delta x) \) (\( n = 0, 1, 2, 3, \ldots \)). In this case, the average scattered light would be constant, that is a modulation would not be detectable. Fortunately, this is never observed since the probability of having identical particles with fixed separation in the measuring volume is very low. Furthermore, the intensity distribution in the measuring volume is not constant because of the Gaussian intensity distribution of the laser beams so that even identical particles will never scatter the same amount of light, which is the basic requirement for total destructive superposition (Rack [8, 11], Duerst et al. [5]). It is more probable that two or more signals of different size and random spacing in the measuring volume will result in signals with phase jumps which are sufficiently strong for LDA evaluation.

Actually, the information of a LDA-system is not distorted by multiple particle signals. Nevertheless, a single particle evaluation, which can be obtained by considering the particle number concentration, simplifies the frequency determination and is preferable.

2.3 Directional Sensitivity

Laser Doppler anemometers, in simple versions, cannot differentiate whether or not particles are crossing the measuring volume in a fixed direction. Crossing the measuring volume in opposite directions makes no difference to the signal generated. It is, therefore, very difficult to apply these simple version LDA-systems to complex flow configurations, for example those with recirculation regions. To resolve this problem of directional ambiguity, frequency shifting electro-optical components can be used. These components are placed in the path of one or both partial beams and change the frequency of the incident light by a very small amount. Because of the small frequency difference of the laser beams, the interference pattern moves. Particles whose direction of propagation is the same as the direction of the fringe movement will yield lower signal frequencies than those particles passing the measuring volume against the fringe pattern movement. From the absolute value of the detected frequency the direction of the particle movement to the LDA-system can be deduced.

To achieve a frequency shift in laser Doppler anemometry, opto-acoustic modulators, Bragg cells, have been used to advantage in the past. A Bragg cell shifts the frequency of a laser beam which crosses the cell at an angle \( \theta \) by an amount of \( k \cdot f_e \), with \( k \) denoting the order and \( f_e \) the exciting frequency of the opto-acoustic cell. The laser beam is diffracted by moving variations in density, of frequency \( f_e \), within a crystal medium. The diffracted laser beam leaves the Bragg cell, with its frequency shifted, at angle \( \phi \). The angle \( \phi \) outside the cell can be computed from the Bragg condition inside the medium. To regain the original direction of propagation, the beam path must be cor-
rected by wedges. Figure 6 shows a comprehensive sketch illustrating the use of Bragg cells for dual beam anemometers. Normally, the detected signal frequencies are in the lower megahertz range. Since Bragg cells are usually driven by an excitation frequency of 40–120 MHz, the use of two Bragg cells is recommended. Thus, a small difference in frequency between the laser beams can be realized whilst avoiding a frequency subtraction by additional electronic components. An additional advantage in using a dual Bragg cell arrangement is that the relative intensity of both partially illuminating laser beams will remain unchanged. The frequency difference of both Bragg cells must always be greater than the maximum expected Doppler signal frequency of particles moving in the direction of shift. When using Bragg cells, we obtain for the velocity according to Eq. (24),

\[ u = \frac{(\Delta f + \Delta f_R) \lambda}{2 \sin \varphi} \]  

where \( \Delta f \) is the Doppler signal frequency and \( \Delta f_R \) is the difference of the excitation frequencies, \( f_{E1} - f_{E2} \), of both Bragg cells.

As an alternative to the use of Bragg cells, a rotating grating can be used. The rotating grating diffracts a laser beam into different partial beams, orders, which have a frequency difference, \( \Delta f_R \), from the undiffracted beam.

As with the use of Bragg cells, the directional deviation from the original can be corrected by small wedges.

### 2.4 Two and Three-Dimensional LDA-Systems

An LDA-system with an additional frequency shifting device allows the measurement of one component of a three-dimensional velocity vector. Many flow problems can be reduced to an investigation in a plane, that is a two-dimensional or even a one-dimensional problem. When the investigations are restricted to mean or integral values, a three-dimensional problem can be covered by a one-dimensional LDA-system by carrying out measurements successively in three different planes. If velocity fluctuations and correlations must be determined, a two or three dimensional, simultaneous measurement of the velocity components is required.

![Fig. 6: Use of opto-acoustical modulators (Bragg cells) for directional sensitivity of laser Doppler systems.](image)

The simultaneous measurement of two or three velocity components can be realized by different optical systems. For two-dimensional investigations, two component, single colour
systems with a polarization and/or frequency range separation (Nett et al. [12], Lourezo et al. [13]) and two component, dual colour systems (e.g. Snyder et al. [14], TSI [15], DISA [16]) can be used. Common to all these systems is that three or four laser beams with different properties, totally or in pairs, such as polarization and frequency, are focused at different angles or in two perpendicular planes inside the measuring volume. Such systems normally represent a combination of two single component systems in two differently orientated planes. Figure 8 shows a two component, single colour and a two component, dual colour system.

With two component LDA systems, the separation of the scattered light signals on the detection side is achieved either by colour or polarization separation or frequency range discrimination. In the case of triple beam, two component LDA-systems the determination of both velocity components has to be carried out by a simple trigonometrical transformation, since the resulting frequencies do not come from orthogonally orientated measuring planes. A more recent method for measuring two velocity components utilizes an electro-optical modulator which switches the measuring plane during the residence time of a particle within the measuring volume (EPRI [17]). Thus, parts of the signal of both measuring planes can be registered and evaluated. LDA-systems for measuring three velocity components are more complex than the two-dimensional arrangements. Normally, the third velocity component, the "on-axis" component, is much more difficult to resolve because its orientation is often parallel to the fringe pattern of the two-dimensional arrangement. Nevertheless, a variety of 3D-LDA-systems has been realized in the past. Hallermair [18] set up an arrangement, which used three laser beams of one colour with different frequency shifts to form a measuring volume. The illuminating laser beams form an equilateral triangle, which allows the determination of all three velocity components with only one detector. The evaluation of the signal frequency must be carried out in a frequency selective manner. Dubnisch et al. [19] realized, in 1976, an arrangement for measuring the three orthogonal velocity components with three laser beams which passed the measuring volume twice due to reflection of the beams. Other versions make use of four laser beams, which form a measuring volume in three non-orthogonal measuring planes (Johansson et al. [20]). These systems, as well as the five beam systems, (TSI [21], DISA [16]) operate with two laser colours and different frequency shifts, which allows realization of three moving interference fringe patterns of different orientation. A method, which allows the simultaneous detection of the three Doppler frequencies by three detectors, which are located differently in space, was presented by Sato et al. in [22]. Miller [23] showed, that a 3D-LDA-system can be realized by three differently orientated reference beam arrangements. Finally, for the determination of three velocity components, combinations of different arrangements can be used, for example, a two component-dual colour system combined with a reference beam arrangement for measuring the third velocity component on the system axis.

3 Signal Processing

In laser Doppler anemometry, the following signal processing methods are usually employed:

Transient recorders
The electrical analogue signals, generated by photodetection elements, are digitized by a transient recorder, connected by interface to a computer which performs the frequency determina-
tion. A transient recorder is a fast analogue to digital converter which is actually characterized by its sampling rate and memory length and width. The selection of such recorders depends on the expected signal frequency range. It should be ensured that the sampling rate is a multiple of the signal frequency sufficient to achieve a resolution of an LDA burst. Transient recorders are suitably used in high as well as in low particle concentration situations. The maximum processable signal frequency range is about 50–100 MHz. Transient recorders process intermittent signals.

Tracker

Use of a frequency tracker is peculiar to situations of high particle concentration in the fluid. The band-pass filtered Doppler signal is compared with the frequency of a voltage controlled oscillator. The oscillator is readjusted to make the difference of both frequencies zero. The voltage adjustment of the oscillator can be used as a measure of the Doppler frequency. Trackers can be used for noisy signals and signal frequencies up to 15 MHz. Multiple particle signals, as well as intermittent signals due to low particle rates, can lead to errors in the determination of time averaged quantities.

Counters

LDA counters work on the basis of time measurement between two events. The time measurement is performed by pulse counting of a high frequency oscillator. The number of cycles which occur within the measuring time are registered by counting the zero crossings of the high-pass filtered LDA-signal. The Doppler frequency can be inferred by division of the number of cycles by time. Typically 8–10 zero crossings are evaluated. Counters are preferred in measuring situations with low signal rates. Intermittent signals up to 100 MHz can be processed. As with tracker processing, counter-based data processing requires better signal quality.

Photon correlation

Photon correlation techniques are used in laser Doppler anemometry especially to resolve noisy signals. The velocity determination is based on autocorrelation of the signal which consists of a train of electron pulses. These pulses are generated by the photon-electron conversion at the photodetector cathode and the subsequent electron amplification due to secondary emission in the dynodes of the photomultiplier used. Generally, the determination of the autocorrelation function can be carried out in an analogue or digital way. The autocorrelation function is a measure for the correlation of the identical signal, spaced in time by a time delay, \( \tau \).

The autocorrelation can be obtained from a single particle signal as well as from contributions from a series of signals to obtain mean velocity and turbulent information. From the period length of the autocorrelation function, the mean frequency of the LDA-signal can be calculated. The turbulence intensity can be deduced from the decay of the amplitude of the autocorrelation function. Normally, many signals are needed to obtain one correlation curve, which implies that real time information of the individual signals is lost. Furthermore, to gain turbulent information is difficult because the shape of the probability density function of the turbulence must be known a priori. Photon correlation techniques are employed with signal frequencies up to 50 MHz.

In Figure 9 a summary of the most important characteristics of the signal processing methods described is given in relation to their application.

In laser Doppler anemometry the errors are mostly due to the signal processing technique and the evaluation algorithms (Ruck et al. [24]). Ensemble averages and time averages should be clearly distinguished since both quantities can be different in turbulent, fluctuating flows with low particle concentrations. With constant, low particle concentration flow intervals of high velocity will contribute relatively more signals to the registered mean value than low velocity flow intervals ("biasing", e.g. Erdmann et al. [25]). This problem can be minimized by increasing the particle concentration so that, statistically seen, the time interval between the signal detection is not dominated by the particle arrival rate but only by the processing time. Hence, it should be ensured that the processor can receive signals whenever it is ready to accept.

Another information weighting in laser Doppler anemometry can be found in signal triggering during the evaluation. By triggering, a pre-selection of signals is performed, which favours LDA information coming from particles of certain size classes. This is due to the fact that different size classes correspond to different signal amplitudes. In laser Doppler anemometry, the fluid velocity is deduced from measured particle velocities, therefore, the knowledge of the particle size classes contributing to the final signals is very important. When measuring turbulent fluctuations especially, information from big and inert particles, which certainly do not follow exactly the continuous phase should be avoided. Using a combined particle size and velocity measuring system (Ruck [7]), systematic investigations of thousands of simultaneous size and velocity signals revealed the influence of signal triggering on the resulting LDA information. It was shown that a higher trigger level on unfiltered LDA-signals

<table>
<thead>
<tr>
<th>Application</th>
<th>Liquid flows (high particle rates)</th>
<th>Gas flows (low particle rates)</th>
<th>Noisy LDA-signals</th>
<th>Real-time information</th>
<th>LDA frequency range up to</th>
<th>Data rate**</th>
<th>Accuracy*** (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient recorder</td>
<td>X</td>
<td>X</td>
<td>(x)*</td>
<td>X</td>
<td>50 MHz</td>
<td>medium</td>
<td>&lt; 0.5%</td>
</tr>
<tr>
<td>Tracker</td>
<td>X</td>
<td>(x)</td>
<td></td>
<td>X</td>
<td>15 MHz</td>
<td>high</td>
<td>1%</td>
</tr>
<tr>
<td>Counter</td>
<td>(x)</td>
<td>X</td>
<td></td>
<td>X</td>
<td>100 MHz</td>
<td>high</td>
<td>1%</td>
</tr>
<tr>
<td>Photon correlator</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>50 MHz</td>
<td>very low</td>
<td>2-3%</td>
</tr>
</tbody>
</table>

Fig. 9: Application of LDA signal processing methods.

(*) limited application
(* *) software-dependent
(**) hardware-dependent
(*** ) frequency-dependent
favours the information from large particles. On the other hand
a higher trigger level on high-pass filtered LDA-signals results in
a preference of the information from small particles (Ruck [26]).
This relationship must be understood as a multiplication of the
scattered response curve with the characteristic modulation
depth, both of which are particle size dependent. Figure 10
shows, for a given median particle size, the apparent median in
both cases.

![Graph showing LDA signal triggering and its influence on the effective particle size distribution.](Image)

Fig. 10: LDA signal triggering and its influence on the effective particle size distribution.

The results show that triggering on high-pass filtered LDA-
signals satisfies the requirement of getting velocity information
from the small particles. Furthermore, criteria for particle
seeding with respect to particle size distribution and concentration
can be deduced from these results (Ruck and Durst [27]).

4 Basic Formulas for Computation

Many applications in laser Doppler anemometry require an
adaptation of the LDA-system to the test section. Often, a
modification of the light beam path must be realized by means
of optical components. In the following chapter, basic formulas
for the lay-out of optical arrangements and for the laser beam
propagation are given.

4.1 Laser Beam Propagation

LDA-systems normally use continuous wave lasers working in the
TEM₀₀ mode. This mode is characterized by a Gaussian
intensity distribution in the laser beam cross section.

\[
I(r,z) = \frac{2P_L}{\pi s^2(\lambda)} \exp\left[-\frac{2r^2}{s^2(\lambda)}\right].
\]

Here, \( r \) denotes the variable radius, \( s \) the laser beam radius, \( P_L \)
the light power of the laser. The laser beam radius, \( s \), is defined
as the distance from the centre of the beam at which the intensity
has fallen by a factor \( 1/e^2 \). Because of the inherent divergence
of a laser beam, its radius at a distance, \( z \) from the laser source,
which has radius, \( s_m \), must be computed using:

\[
s(z) = s_m \left[1 + \left(\frac{2z}{\lambda s^2(\lambda)}\right)^2\right]^{1/2}. \tag{27}
\]

Due to the laser beam generation in an optical resonator and the
resulting Gaussian intensity distribution, the beam propagation
does not obey the rules of geometrical optics. In laser Doppler
systems, plane wave fronts must be achieved within the measur-
ing volume. Plane wave fronts always occur in the waist of a laser
beam and so the lay-out of the system must incorporate suc-
cessive imaging of the original laser waist up to the measuring
volume. The propagation of Gaussian light beams through a lens
can be computed using the relations given in Figure 11.

![Diagram showing Gaussian beam propagation through a lens.](Image)

Generally, imaging of the \( n \)-th waist to the \((n+1)\)-th waist by a lens can be computed:

\[
s_{n+1} = s_n \frac{f_B}{f_B - f_B} \left[\frac{z_n - f_B}{z_n - f_B} + \frac{f_B}{f_B - f_B}\right]^{1/2} \tag{28}
\]

\[
z_{n+1} = f_B \left[\frac{z_n - f_B}{f_B - f_B} + \frac{f_B}{f_B - f_B}\right]^{1/2} \tag{29}
\]

Fig. 11: Gaussian light beam propagation through a lens.

Typical differences from geometrical optics occur if the waist
distance approaches the focal length. In this case, the focal point
on one side is imaged into the focal point of the other side of the
lens.

Using Eqs. (27) to (29) allows calculation of the beam radius for
any position of the beam path in the optical system. For a spacing \( \Delta x \), the number of fringes inside the measuring volume can
now be computed with:

\[
N' = \frac{2s \cos \varphi}{\Delta x}. \tag{30}
\]

4.2 Scattered Light Distribution

The spatial distribution of light scattered by particles is complex
and is the sum of different components, generally diffraction,
refraction and reflection. In forward scattering the diffracted
portion dominates. A comprehensive analytical treatment was
first given by Mie [28]. His theory was deduced for spherical par-
cicles and plane wave fronts, (see also van de Hulst [29] and
Kerker [30]). In Figure 12, the spatial distribution of scattered
light, for a water droplet of 1 \( \mu \)m in diameter, is given for two dif-
ferent states of polarization.

The scattered intensities between forward and backward direc-
tions can differ by orders of magnitude. It is, therefore, advan-
tageous if the scattered light is detected in the forward direction
because it can be more than a factor of \( 10^5 \) stronger than in the
backward direction. Measurement in the backward direction, usual-
ly requires higher laser power.
depends on $d^6$. The transitional region, sometimes referred to as the Mie range, is characterized by fluctuations of the response curve. The fluctuations are strongly dependent on the optical properties of the particles and on the direction and solid angle of the detection system. The transitional region extends up to a particle size of several $\mu$m in diameter. For bigger particles the scattered power is determined by the projection area of the particles and the rules of geometrical optics apply. The computation of the scattering efficiency is normally performed by computer programs, which mostly but not exclusively are based on Mie theory (Cherdron et al. [31], Gréhan et al. [32], Durst and Ruck [33], Richter [34]).

5 Application of LDA in Single and Multiphase Flows

Laser Doppler anemometry can be applied to velocity measurements in single phase as well as in two- or multiphase flows. Experimental investigations in single phase gas flows can usually be achieved very easily. In the case of two- or multiphase flows, the application of the LDA principle is often more difficult because of problems caused by the interfaces of the test section. Here, one has to consider the refraction of the laser beams and the local displacement of the measuring control volume. For the general case of having three different media involved, with different refractive indices, the displacement of the measuring volume can be deduced as indicated in Figure 14. Although the laser beams propagate in another medium at another angle before entering the interface, the fringe spacing according to Eq. (22) does not change. The change of angle in Eq. (22) is compensated by a corresponding change of wavelength, so that the original fringe spacing will hold for all the successive media. LDA measurements in flow regions with cylindrical, curved or irregular interfaces require a specific correction of the measuring volume displacement which cannot be given in universal formulae (for measurements in cylindrical tubes, see Thompson et al. [35]).

Application of laser Doppler anemometry to multiphase investigations often requires discrimination of the signals coming from different phases. It is known that LDA signals are generated whenever interfaces pass the measuring control volume.

\[
\begin{align*}
\sin \varphi_1 &= n_1 \sin \varphi_1 = n_2 \sin \varphi_2 = n_3 \sin \varphi_3 \\
x_w &= \frac{a}{2 \tan \varphi_3} \\
x &= \frac{A}{2 \tan \varphi_3} \\
A \text{ AND } \varphi_1 \text{ CAN BE MEASURED OR ARE KNOWN FROM LDA SYSTEM SPECIFICATIONS} \\
a &= A - 2b \\
b &= D \tan \varphi_1 = \frac{D \sin \varphi_1}{[1 - \sin^2 \varphi_1]^{1/2}} \\
b &= \frac{D n_1 \sin \varphi_1}{n_2 \left[1 - \left(\frac{n_1}{n_2} \sin \varphi_1\right)\right]^{1/2}} \\
(5.6) \text{ AND (5.4) IN (5.2) YIELDS} \\
x_w &= f(n_1, n_2, n_3, \varphi_1, D, A)
\end{align*}
\]
Figure 15 sketches the unfavourable situations which are sometimes found in multiphase flows. For many experimental investigations, for example erosion and corrosion studies, deposition and transport mechanisms, the particulate phase of a multiphase flow is the primary interest. The resulting LDA signals must be properly attributed to the separate phases. This can be achieved by discrimination procedures, for example by amplitude selection or by signal shape.

In multiphase flows, signals can originate from phase regions which have greater dimensions than the measuring volume itself. Even in this case, laser Doppler anemometry can be applied as long as the transparency of the region investigated is ensured. As shown by Durst and Zore [36], Wigley [37], Srinivasan et al. [38], Martin et al. [39], Brankovic et al. [40], multiphase flows can be investigated by laser Doppler anemometry for bigger particle dimensions and for bubble flows. The LDA signal shape is different for small particles and for particles bigger than the measuring volume. For transparent media a triple signal is registered in forward scattering whereas a nontransparent particle yields a double signal. In backward light scattering, both particle types generate a single burst.

6 LDA-Probe Systems

The versatility of instrumental arrangements for LDA systems has been extended in recent years by the use of glass fibre probe systems. The motivation for the development of such systems was to get rid of large support units and to create an arrangement which combines the advantages of laser Doppler anemometry with the flexibility of conventional probe systems. Furthermore, the number of possible applications for the LDA technique was expected to increase, for example its use in fire hazardous situations.

At the beginning of this development, different types of fibres were tested for transmission of coherent light of a sufficient quality. Two types of fibres, monomode and gradient index fibres, were found to be suitable for this purpose. The best conservation of coherence is achieved by monomode fibres which have a diameter of a few μm and ensure that only one mode is transmitted. Another fibre type, the gradient index fibre which is used in telecommunications, could also be used to transmit coherent light but with certain limitations. The first LDA systems based on light wave guides were presented by Danel [41] and Dyott [42]. The use of gradient index fibres, combined with an optimized light coupling in order to minimize the number of modes, leads to LDA probe systems with fibre transmission distances up to 200 m (Ruck et al. [43], Nett [44], Ruck [45], Durst et al. [46]). Currently for shorter transmission distances, monomode fibres are mainly used. Due to the very small diameter of the monomode fibre, the adjustment can be disadvantageous but the fibre has better optical properties such as single mode transmissions or the preservation of polarization (Baie [47], So [48], TSI [15], DISA [16]). In Figure 16, the development steps of fibre-based LDA systems are shown. Presently probe systems are being developed, which allow two-dimensional, direction-sensitive measurement by backward light scattering. The miniaturisation of LDA probes could be achieved, in the case of two-dimensional arrangements, by separating the probe and the optical components such as the beam splitter and the frequency shifting device. All these necessary components are located before the light input coupling which facilitates the construction of the probe.

Fig. 15: LDA application in multiphase flows.

Fig. 16: Development of fibre-based LDA systems.

Fig. 17: Two-dimensional, direction-sensitive LDA probe system, IfH University of Karlsruhe.
In this case, the probe consists only of the housing, a lens combination and fibre mounts. In Figure 17 is shown a recent development of the two-dimensional, direction-sensitive LDA probe system for backward light scattering. As an alternative to the development of the relatively complex probe systems described, simple and cheap LDA probes have been constructed. This development was stimulated by the need to have simple probes which can be applied to multiple control situations in industry. For these applications, the quantitative determination of velocity in fluidic systems, without directional information, suffices in many cases. Figure 18 gives a sketch of different simple LDA probe concepts. The detection of the scattered light can be carried out by an additional detection probe in the forward direction or, as we have seen before, in the backward direction achieving detection with the same probe.

![Diagram of LDA probe concepts](image)

**Fig. 18:** LDA simple probe concepts.

7 Final Remarks

There is no doubt, that laser Doppler anemometry has proved to be an advantageous measuring technique for fluid velocity and that the use of the technique is increasing. Applications encompass complex single or multiphase flow investigations with separations to flow analysis in combustion chambers and the monitoring of multiple flow processes on an industrial scale. The development of fibre-based LDA probe systems has improved and extended the applicability of the LDA technique, for example to hazardous measuring sites. The ongoing trend towards miniaturisation of LDA measuring systems will lead to two-dimensional or three-dimensional probe systems in the years ahead. In the future, the use of semiconductor lasers will play an important part. It is uncertain whether the efforts to further miniaturise LDA systems will continue since the costs are presently comparable to conventional LDA systems. The prime costs are likely to increase drastically with further miniaturisation. Simple and cheap LDA probes have great potential to be widely used as multiple sensors in industrial production processes.

8 Acknowledgements

The author gratefully acknowledges the financial support which was given to parts of this research by the Deutsche Forschungsgemeinschaft under grant No. EI/SFB 210 and Ru 345/2. Thanks are also due to Mrs. D. Bring, who helped to finalise this report.

9 Symbols and Abbreviations

\( a \) geometrical extension  
\( A \) distance  
\( b \) geometrical extension  
\( \dot{c} \) velocity of light  
\( d \) particle diameter  
\( D \) thickness  
\( E \) electric field  
\( f_0 \) laser light frequency  
\( f^* \) Doppler shifted light frequency, emitter  
\( f_0 \) Doppler shifted light frequency, detector  
\( f_e \) driver frequency for bragg cells  
\( \Delta f \) Doppler beat frequency (signal)  
\( \Delta f_b \) difference frequency of bragg cells  
\( \Delta f_r \) difference frequency with respect to zero order beam  
\( I \) intensity  
\( I \) light intensity  
\( k \) \( 2\pi/\lambda \), constant  
\( r \) direction vector  
\( f_d \) direction vector of detection  
\( n \) refractive index, number  
\( N \) number of lines  
\( N_b \) number of fringes  
\( P_l \) laser power  
\( r \) variable radius  
\( l \) scattering intensity  
\( s \) \( 1/e^2 \) = laser beam radius  
\( S \) line spacing  
\( t \) time  
\( T \) integration time  
\( \dot{u} \) velocity vector  
\( u_\perp \) velocity component perpendicular to bisector of crossing beams  
\( w \) substitution constant  
\( x \) distance  
\( \Delta x \) fringe spacing  
\( y \) distance  
\( z \) distance  
\( \delta \) angle of beam propagation  
\( \eta \) modulation depth (visibility)  
\( \theta \) angle of beam propagation  
\( \lambda \) laser light wavelength
\[ \lambda \]
ultrasonic wavelength (bragg cell)

\[ \pi \]
3.141596 ....

\[ \varphi \]
semi-angle of crossing beams

\[ \psi \]
angle between direction of velocity vector and bisector of crossing beams

\[ \omega \]
2 \( \pi \cdot f \)

10 References


